A Purely Quantum Mechanical Equation for the Magnetic Monopole

The basic derivation is in the SI units of the Ampere-meter convention.

Maxwell’s equations including the magnetic monopole term in Faraday’s law are:

\[-\nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t} + \mu_0 \vec{j}_m\]

\[\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{j}_e\]

\[\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0}\]

\[\nabla \cdot \vec{B} = \mu_0 \rho_m\]

Starting with the divergence of the Poynting vector:

\[\nabla \cdot \vec{E} \times \vec{B} = \]

\[= \vec{B} \cdot \nabla \times \vec{E} - \vec{E} \cdot \nabla \times \vec{B}\]

\[= \vec{B} \cdot \left(-\frac{\partial \vec{B}}{\partial t} - \mu_0 \vec{j}_m\right) - \vec{E} \cdot \left\{ \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{j}_e \right\} \quad \text{Substituting } \nabla \times \vec{E} \text{ and } \nabla \times \vec{B}\]

\[= -\vec{B} \cdot \left(\frac{\partial \vec{B}}{\partial t} + \mu_0 \vec{j}_m\right) - \vec{E} \cdot \left\{ \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{j}_e \right\} \quad (Eqn 1)\]

Remember Heisenberg’s equations of motion are:

\[[H, \vec{B}] = i\hbar \frac{\partial \vec{B}}{\partial t}\]

\[[H, \vec{E}] = i\hbar \frac{\partial \vec{E}}{\partial t}\]

Therefore we want to transform equation 1 into a simpler form by making the following transformations from a complex equation of motion to a simpler one:
\[
\frac{\partial \vec{B}}{\partial t} = -i\frac{1}{\hbar}[H, \vec{B}] \Rightarrow -i\frac{1}{\hbar}[H, \vec{B}] - \mu_0 \vec{J}_m
\]
\[
\frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} = -i\frac{1}{\hbar c^2}[H, \vec{E}] \Rightarrow -i\frac{1}{\hbar c}[H, \vec{E}] - \mu_0 \vec{J}_e
\]

This mapping is a very interesting and fascinating mapping by the way.

Then finally the desired equation is obtained:

\[
\vec{\nabla} \cdot \vec{S} = -\vec{B} \cdot \frac{-i}{\hbar}[H, \vec{B}] - \vec{E} \cdot \frac{-i}{\hbar}[H, \vec{E}]
\]
\[
- i\hbar \vec{\nabla} \cdot \vec{S} = \vec{E} \cdot [H, \vec{E}] + \vec{B} \cdot [H, \vec{B}] \quad (Eqn \ 2)
\]

Here I started out with Maxwell’s equations with the magnetic monopole term included in Faraday’s law. Ampere’s law remains the same. That is the first two lines.

Next take the divergence of the cross product of the electric and magnetic fields, or the divergence of the Poynting vector. Next express the changing magnetic and electric fields as Heisenberg’s equations for motion. After a transformation or mapping we get:

\[
\vec{p} \cdot \vec{S} = \vec{E} \cdot [H, \vec{E}] + \vec{B} \cdot [H, \vec{B}] \quad \text{Alternatively, the following equation obtained:}
\]
\[
- i\hbar \vec{\nabla} \cdot \vec{S} = \vec{E} \cdot [H, \vec{E}] + \vec{B} \cdot [H, \vec{B}]
\]

This last equation is the Schrödinger-Heisenberg form of the equation. I have used the two correspondence principles: classical mechanics through Heisenberg’s equation of motion and electromagnetic theory through Maxwell’s equations (It is rumored that Vladamir Fock, Russia’s famous physicist always used a correspondence principle between electromagnetism and quantum theory). Of course Heisenberg originally obtained his equations of motion through the pure data and energy level shifts and spectral data. However, H is the classical Hamiltonian. The H can also be a relativistic Hamiltonian of course. The Schrödinger part is through \(\vec{p} \rightarrow -i\hbar \vec{\nabla} \).

The ideas that I had for calculations to obtain any knowledge of magnetic monopole terms would be by adjusting the H, the Hamiltonian and the decomposition of the electric and magnetic fields. One way to go would be to express the \(H\), the Hamiltonian with creation and annihilation operators: \(a^*, a\) for bosons and \(b^*, b\) for fermions. As J.D. Jackson’s Electromagnetic Theory text says, for a magnetic monopole it may be fruitful to include a covariant derivative. Then following this we could say the Hamiltonian is as follows

\(H = \hbar \alpha a^* + \lambda (-i\hbar \vec{\nabla})\). Maybe the second term being a covariant derivative like Jackson’s text says should be there. Of course, with the \(\lambda\), the units should turn out to be that of energy. Also we wonder if the monopoles turn out to be fermions or bosons. We can consider an N-S dipole magnet as a bound state of an N-pole and an S-pole of magnetic charge. Therefore, whether a monopole alone is a fermion or a boson, its bound state as a normal magnet will be the addition of the two spins. One question arises: how do you effect the decomposition of the magnetic monopole field. Is it basically the same as the decomposition of the electric field by
photons of two different polarizations? The magnetic monopole quickly coalesces into the magnetic dipole of a North and South Pole, probably quicker than an electric dipole forms. We can have the following idea for the decomposition of the magnetic monopole existing by itself as a N-pole and a S-pole.

\[ m_{\text{photon}} \rightarrow \sum \{ s(+)a * + s(-)a \} \]

Or we can have the idea:

\[ m_{\text{photon}} \rightarrow \sum \{ s(+)b * + s(-)b \} \]

Of course the usual decomposition of the electric field is:

\[ e_{\text{photon}} \rightarrow \sum (\hbar \omega_+ + \hbar \omega_-) \]

Also these transformation equations are very interesting in and of themselves. Maybe they hold key to a transformation to the monopole itself. After doing the transformation it brings the equation into the classical electromagnetic field. That is, the divergence of the Poynting vector equals the time-rate of change for the energy density of the electric and magnetic fields. Therefore, the opposite transformation leads to the electric and magnetic monopole charge present.

\[ \frac{\partial \vec{B}}{\partial t} = -\frac{i}{\hbar} [H, \vec{B}] \Rightarrow -\frac{i}{\hbar} [H, \vec{B}] - \mu_0 \vec{j}_m \quad (Eqn 3) \]

\[ \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} = -\frac{i}{\hbar c^2} [H, \vec{E}] \Rightarrow -\frac{i}{\hbar} [H, \vec{E}] - \mu_0 \vec{j}_e \quad (Eqn 4) \]

Therefore, note this transformation down as very interesting of a result. My professor Ernest Thieliker at the University of South Florida always showed me abstract spaces for quantum mechanics while I was still an undergraduate. I must credit him for instilling in my young mind such mathematics of quantum mechanics.

Lastly, like Dirac’s equation was relativistic and quantum mechanically correct he shed more light on the electron with it. He also predicted the fact that antimatter exists in the universe. However, the antimatter part of Dirac’s equation was only verified 6 years later.

So my equation may be able to explain many items to look for in high energy physics as we look for the magnetic monopole itself. Finding the magnetic monopole may help explain fractional charges, or fractions of the electron charge which we thought were the quantum of charge.
The two transformations revisited

\[
\frac{\partial \vec{B}}{\partial t} = -\frac{i}{\hbar} [H, \vec{B}] \Rightarrow -\frac{i}{\hbar} [H, \vec{B}] = \mu_0 \vec{j}_m
\]

\[
\frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} = -\frac{i}{hc^2} [H, \vec{E}] \Rightarrow -\frac{i}{hc^2} [H, \vec{E}] = \mu_0 \vec{j}_e
\]

Let us assume that there exist two transformations \(\Lambda_m\) and \(\Lambda_e\). We can assume they are linear transformations and they are some kind of translations operators. Also we must consider their commutations relations.

What are these conditions? Eventually we hope to answer these questions.

\[
[\Lambda_m, H] \quad [\Lambda_m, \vec{B}] \quad [\Lambda_m, [H, \vec{B}]]
\]

What are these conditions?

\[
[\Lambda_e, H] \quad [\Lambda_e, \vec{E}] \quad [\Lambda_e, [H, \vec{E}]]
\]

In general, we have:

\[
\Lambda_T [H, \vec{B}] = [H, \vec{B}] + \vec{\pi}_m \quad (Eqn \ 5)
\]

\[
\Lambda_T [H, \vec{E}] = [H, \vec{E}] + \vec{\pi}_e \quad (Eqn \ 6)
\]

Plus \(\Lambda_T^{-1}\) exists and therefore \(\Lambda_T^{-1} \Lambda_T = I\) also is valid. One obvious thing may be about \(\Lambda_T\) is that it acts on vectors or scalars, \(\Lambda_T \vec{A}\) and \(\Lambda_T B\).

Therefore, beginning we start with the Faraday’s law and the magnetic field with the magnetic monopole term added. Then we will do Ampere’s-Maxwell’s law and the electric field where no magnetic monopole field correction is needed (at present). The resulting two equations with the Lambda, \(\Lambda_T\) transformation wrapped around them are:
\[ \frac{\partial \vec{B}}{\partial t} = -\frac{i}{\hbar} [H, \vec{B}] \Rightarrow -\frac{i}{\hbar} [H, \vec{B}] - \mu_0 \vec{j}_m \]
\[
-\frac{i}{\hbar} \Lambda_m [H, \vec{B}] = -\frac{i}{\hbar} [H, \vec{B}] - \mu_0 \vec{j}_m
\]
\[
-\frac{i}{\hbar} \Lambda_m [H, \vec{B}] + \frac{i}{\hbar} [H, \vec{B}] = -\mu_0 \vec{j}_m
\]
\[
-\frac{i}{\hbar} [\Lambda_m - 1] [H, \vec{B}] = -\mu_0 \vec{j}_m
\]
\[
-\frac{i}{\hbar} I [H, \vec{B}] = -\mu_0 [\Lambda_m - 1]^{-1} \vec{j}_m
\]
\[
i\hbar \frac{\partial \vec{B}}{\partial t} = [H, \vec{B}] = -i\hbar \mu_0 [\Lambda_m - 1]^{-1} \vec{j}_m
\]
\[
\frac{\partial \vec{B}}{\partial t} = -\mu_0 [\Lambda_m - 1]^{-1} \vec{j}_m
\]

Therefore:
\[ \nabla \times \vec{E} = \mu_0 ([\Lambda_m - 1]^{-1} - 1) \vec{j}_m \quad (Eqn \ 7) \]

This is Faraday's law with the magnetic monopole term included and with the Lambda, \( \Lambda_m \) transformation incorporated into it. This is no doubt, a very interesting result for one of Maxwell's equations when including the magnetic monopole term. Then the task is to find this linear translation operator \( \Lambda_m \). In addition, we assume \( \Lambda_m^{-1} \) exists also and \( [\Lambda_m - 1]^{-1} \) and \( [\Lambda_m - 1] \) exists as well and therefore, \( [\Lambda_m - 1][\Lambda_m - 1]^{-1} = I \) is valid.

Likewise for Ampere's-Maxwell's law we get:
\[
\frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} = -\mu_0 [\Lambda_e - 1]^{-1} \vec{j}_e
\]
\[
\nabla \times \vec{B} = -\mu_0 ([\Lambda_e - 1]^{-1} - 1) \vec{j}_e \quad (Eqn \ 8)
\]

You will notice even without the magnetic monopole term in Ampere's law it will nevertheless transform by \( \Lambda_T \). The Lambda transformation wrapped around Ampere's-Maxwell's law in no way violates neither the original Ampere's law nor the Maxwell's original equation. However, Faraday's law remains invariant under the \( \Lambda_T \) transformation. We have then, not including the magnetic monopole term:
\[ \nabla \times \vec{B} = -\mu_0 ([\Lambda_e - 1]^{-1} - 1) \vec{j}_e \quad \text{And} \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]
Then applying Stoke’s theorem we obtain:

\[
\iint_{S} \nabla \times \mathbf{B} \cdot d\mathbf{S} = -\mu_{0} \iint_{S} \left( [\Lambda_{e}^{-1} - 1] \mathbf{j}_{e} \right) \cdot d\mathbf{S} \quad \text{(Eqn 9)}
\]

\[
\oint_{C} \mathbf{B} \cdot d\mathbf{s} = -\mu_{0} \iint_{S} \left( [\Lambda_{e}^{-1} - 1] \mathbf{j}_{e} \right) \cdot d\mathbf{S} \quad \text{(Eqn 10)}
\]

The usual result for Stoke’s theorem applied to Ampere’s-Maxell’s law is we obtain the true current and the displacement current:

\[
\oint_{C} \mathbf{B} \cdot d\mathbf{s} = \iint_{S} \left( \varepsilon_{0} \mu_{0} \frac{\partial \mathbf{E}}{\partial t} + \mu_{0} \mathbf{j}_{e} \right) \cdot d\mathbf{S} = \iint_{S} \varepsilon_{0} \mu_{0} \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{S} + \mu_{0} \frac{dQ_{\text{true}}}{dt}
\]

Since:

\[
\iint_{S} \varepsilon_{0} \mu_{0} \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{S} = S \varepsilon_{0} \mu_{0} \frac{\partial \mathbf{E}}{\partial t} = S \mu_{0} I_{\text{displacement}} = S \mu_{0} \frac{I_{\text{displacement}}}{S} = \mu_{0} I_{\text{displacement}}
\]

Then we obtain:

\[
\oint_{C} \mathbf{B} \cdot d\mathbf{s} = \iint_{S} \varepsilon_{0} \mu_{0} \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{S} + \mu_{0} \frac{dQ_{\text{true}}}{dt} = \mu_{0} I_{\text{displacement}} + \mu_{0} I_{\text{true}} \quad \text{(Eqn 11)}
\]

Therefore, there is no usual result of the displacement current and the true current according to the common Stoke’s calculation without the Lambda, \( \Lambda_{e} \) transformation wrapped around Ampere’s-Maxwell’s law. Usually you can readily integrate to these two currents. This actually makes more sense. Since when does the electric displacement field disconnect from the system making a separate current. The displacement electric field vector always contributes further to the movement of charged particles in a current! Interesting conclusion: What does this all mean for alternating currents? This new transformation factor multiplying the true current density namely \( [\Lambda_{e}^{-1} - 1] \) is very interesting for electronics and other alternating current circuits and their future breakthroughs concerning electronics. What does the \( \Lambda_{m} \) and \( \Lambda_{e} \) do for alternating circuits? In addition, seeing this new Stoke’s theorem calculation: what does the \( \Lambda_{m} \) and \( \Lambda_{e} \) do for pure mathematics of vector analysis, the Stoke’s, and other theorems? Can Stoke’s theorem be reformulated with the \( \Lambda_{m} \) and \( \Lambda_{e} \) transformations? Just an extra thought about vector analysis. Then finally, comparing Eqn 10 with Eqn 11 we have hopes of obtaining an explicit result for \( \Lambda_{e} \), in the Ampere’s-Maxwell’s law with the Lambda transformation wrapped around it. This is a fundamental change of Classical physics, namely, Classical electromagnetic dynamics. As you can see questions remain and the Lambda, \( \Lambda_{T} \) transformation actually somehow takes the place of \( \mu_{0} \epsilon_{0} \frac{\partial \mathbf{E}}{\partial t} \) in Ampere’s-Maxwell’s law.

Therefore \( \Lambda_{T} \) is time varying and dynamic. The operator \( [\Lambda_{e}^{-1} - 1] \) appears to be of utmost importance for the magnetic monopole, and Ampere’s law.
Then we ask in conclusion is \( \Lambda_T \) a quantum mechanical, dynamical variable or a new transformation law? Is there a difference between the Lambda transformation, \( \Lambda_T \), for Classical as opposed to Quantum Mechanics? We can compare for Classical electromagnetic dynamics the difference between Eqn 10 and Eqn 11 thus hopefully calculate the Lambda transformation, \( \Lambda_T \), in this case. This will be done in another future paper. Also in that future paper we hope to shed light upon the Lambda transformation, \( \Lambda_T \), for transforming Quantum mechanical dynamics.

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